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Solution of Non-Homogeneous Burgers Equation by Haar Wavelet Method

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ABSTRACT: A numerical scheme based on uniform Haar wavelets is used to solve the nonhomogeneous Burgers equation. The quasilinearization technique is used to conveniently handle the nonlinear terms in the nonhomogeneous Burgers equation. The Haar wavelet preliminaries and function approximation are explained in brief. We have solved nonhomogeneous Burgers equation with two initial conditions, namely, linear and periodic initial condition and are presented as two examples. The Haar wavelet solution of these two examples are compared with exact solution and finite difference method solution.

Keywords: Haar wavelets, Non-homogeneous Burgers equation, Quasilinearization, Collocation points, Finite difference.

I. INTRODUCTION

The Haar wavelet is the principal known wavelet and was proposed in 1909 by Alfred Haar. The Haar wavelet is likewise the least complex conceivable wavelet. Over the recent decades, wavelets by and large have picked up a respectable status because of their applications in different disciplines and in that capacity have numerous examples of overcoming adversity. Prominent effects of their studies are in the fields of signal and image processing, numerical analysis, differential and integral equations, tomography, and so on. A standout amongst the best utilizations of wavelets has been in image processing. The FBI has built up a wavelet based algorithm for fingerprint compression. Wavelets have the capability to designate functions at different levels of resolution, which permits building up a chain of approximate solutions of equations. Compactly supported wavelets are localized in space, wherein solutions can be refined in regions of sharp variations/transients without going for new grid generation, which is the basic methodology in established numerical schemes. Sumana and Achala [1] have given a brief report on Haar wavelets.

The proposed technique plans to cover a more extensive range of problems, by applying it to verifiably essential and an extremely helpful class of initial boundary value problems, thereby enhancing its applicability. Chen and Hsiao [2] have effectively used Haar wavelets in their work on optimization theory, control theory, stiff differential equations, and so on, wherein the method is demonstrated convincingly. To exhibit the efficiency and effectiveness of the method, the celebrated non-homogeneous Burgers equations are analyzed for their solutions. The nonhomogeneous Burgers equation is given by,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=\nu\frac{\partial^2 u}{\partial x^2}+F(x,t), 0<x<1, t>0, \tag{1}$$

where ν is the fluid viscosity and $F(x,t)$ is the nonhomogeneous term.

Nonhomogeneous Burgers equation was first derived from the hydrodynamics equation and used in the laser generation of sound. It was also applied to other physical phenomena such as wind forcing the buildup of water waves, electrohydrodyanamic field in plasma physics, and design of feedback control. Karabutov et al. [3] obtained the analytical solution of equation (1) with $F(x,t)=A\sin(x), A>0$. Ding et al. [4] studied the solution of (1) with $F(x,t)=-kx, k>0$. Srinivasa Rao [5] obtained large time asymptotic for solutions of equation (1) with $F(x,t)=kx, k>0$. Recently Srinivasa Rao [6] represented the solutions of equation (1) with $F(x,t)=kx/(2\beta t+1)^2, k>0, \beta>0$. More recently, Sarboland [7] obtained the solution of equation (1) for different $F(x,t)$ by using direct and indirect multiquadratic quasi interpolation schemes.

In the recent years, Haar wavelets have been widely used to solve PDEs. Lepik [8] applied Haar wavelets to solve evolution equations. Shi et. al. [9] solved wave equation using Haar wavelets. Ram Jiwari [10] used Haar wavelets to solve Burgers equation. Hariharan et. al. [11, 12, 13, 14, 15] applied Haar wavelets to solve Cahn-Allen Equation, Fisher's equation, FitzHugh-Nagumo equation, Klein-Gordon equation, Sine-Gordon equation and some nonlinear parabolic equations. Dhawan et. al. [16] solved heat equation using Haar wavelets.

Celik [17] used Haar wavelets to solve generalized Burgers-Huxley equation. Bujurke et. al. [18] applied wavelet-multigrid method to solve elliptic partial differential equations.

An efficient and novel numerical scheme based on uniform Haar wavelets [19] is used to solve the nonhomogeneous Burgers equation. The quasilinearization technique [20] is used to conveniently handle the nonlinear terms in the nonhomogeneous Burgers equation. The paper is organized as follows. The Haar wavelet preliminaries, function approximation and the method of solution are presented in Section 2. The numerical examples and discussions are presented in Section 3. The conclusions drawn are presented in Section 4.

II. HAAR WAVELETS

Haar wavelet family for $x \in [0,1]$ is defined [19] as follows

$$h_i(x) = \begin{cases} 1 & \text{for } x \in [\xi_1, \xi_2) \\ -1 & \text{for } x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

where

$$\xi_1 = \frac{k}{m},\ \xi_2 = \frac{k+0.5}{m},\ \xi_3 = \frac{k+1}{m}. \tag{3}$$

Here $m = 2^n,\ n = 0,1,\ldots,J$ indicates the level of the wavelet; $k = 0,1,\ldots,m-1$ is the translation parameter. J is the maximum level of resolution. The index i in equation (1) is calculated by the formula $i = m + k + 1$. In the case of minimum values $m = 1,\ k = 0$ we have $i = 2$. The maximum value of i is $i = 2M = 2^{J+1}$. For $i = 1$, $h_1(x)$ is assumed to be the scaling function which is defined as follows.

$$h_1(x) = \begin{cases} 1 & \text{for } x \in [0,1) \\ 0 & \text{elsewhere} \end{cases} \tag{4}$$

We require the following integrals in order to solve second order partial differential equations.

$$p_i(x) = \int_0^x h_i(x)dx = \begin{cases} x - \xi_1 & \text{for } x \in [\xi_1, \xi_2) \\ \xi_3 - x & \text{for } x \in [\xi_2, \xi_3) \\ 0 & \text{elsewhere} \end{cases} \tag{5}$$

$$q_i(x) = \int_0^x p_i(x)dx = \begin{cases} \frac{1}{2}(x-\xi_1)^2 & \text{for } x \in [\xi_1, \xi_2) \\ \frac{1}{4m^2} - \frac{1}{2}(\xi_3 - x)^2 & \text{for } x \in [\xi_2, \xi_3) \\ \frac{1}{4m^2} & \text{for } x \in [\xi_3, 1] \\ 0 & \text{elsewhere} \end{cases} \tag{6}$$

Any function $y(x)$ which is square integrable on $(0,1)$ can be expressed in terms of Haar wavelets as follows.

$$y(x) = \sum_{i=1}^{\infty} a(i) h_i(x). \tag{7}$$

Here, the expansion of $y(x)$ is an infinite series. If $y(x)$ is approximated as piecewise constant in each sub-area, then it will be terminated at finite terms, that is,

$$y(x) = \sum_{i=1}^{2M} a(i) h_i(x), \tag{8}$$

where the wavelet coefficients $a(i), i = 1,2,\ldots,2M$ are to be determined.

III. METHOD OF SOLUTION

Consider an initial boundary value problem (IBVP) for the nonhomogeneous Burgers equation (1) with the initial and boundary conditions

$$u(x,0) = f(x), 0 \le x \le 1, \tag{9}$$

$$\left.\begin{aligned} u(0,t) &= g_1(t) \\ u(1,t) &= g_2(t) \end{aligned}\right\} t \ge 0. \tag{10}$$

Let us divide the interval $[0,T]$ into N equal parts of length $\Delta t = T/N$ and denote $t_s = (s-1)\Delta t, s = 1,2,\ldots,N$. We assume a Haar wavelet solution for equation (1) in the form

$$\dot{u}''(x,t) = \sum_{i=1}^{2M} a_s(i) h_i(x), \tag{11}$$

where the dot and prime denote differentiation with respect to t and x respectively, the row vector a_s is constant in the subinterval $t \in [t_s, t_{s+1}]$.

Integrating equation (11) with respect to t in the limits $[t_s, t]$, integrating the resultant equation with respect to x in the limits $[0,x]$, again integrating the resultant equation with respect to x in the limits $[0,x]$, and differentiating the resultant equation with respect to t, we obtain the following equations respectively.

$$u''(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i) h_i(x) + u''(x,t_s), \tag{12}$$

$$u'(x,t) = (t-t_s)\sum_{i=1}^{2M} a_s(i) p_i(x) + u'(x,t_s) - u'(0,t_s) + u'(0,t), \tag{13}$$

$$u(x,t) = \left(t-t_s\right)\sum_{i=1}^{2M} a_s(i) q_i(x) + u\left(x,t_s\right) - u\left(0,t_s\right) + x\left[u'(0,t) - u'\left(0,t_s\right)\right] + u(0,t), \tag{14}$$

$$\dot{u}(x,t) = \sum_{i=1}^{2M} a_s(i) q_i(x) + \dot{u}(0,t) + x\dot{u}'(0,t). \tag{15}$$

Using the boundary conditions (10), we have

$$\begin{aligned} u(0,t_s) &= g_1(t_s), \quad u(1,t_s) = g_2(t_s), \\ \dot{u}(0,t) &= \dot{g}_1(t), \qquad \dot{u}(1,t) = \dot{g}_2(t). \end{aligned} \tag{16}$$

Putting $x = 1$ in equations (14) and (15) and using the conditions in (16), we obtain

$$u'(0,t) - u'(0,t_s) = -\left(t-t_s\right)\sum_{i=1}^{2M} a_s(i) q_i(1) + g_2(t) - g_1(t) - g_2(t_s) + g_1(t_s) \tag{17}$$

$$\dot{u}'(0,t) = -\sum_{i=1}^{2M} a_s(i) q_i(1) + \dot{g}_2(t) - \dot{g}_1(t) \tag{18}$$

The wavelet collocation points are defined as

$$x_l = \frac{l-0.5}{2M}, l = 1,2,\ldots,2M. \tag{19}$$

Substituting equations (16) - (18) in equations (12) - (15) and taking $x \to x_l$, $t \to t_{s+1}$, we get

$$u''(x_l,t_{s+1}) = \Delta t \sum_{i=1}^{2M} a_s(i) h_i(x_l) + u''(x_l,t_s), \tag{20}$$

$$u'(x_l,t_{s+1}) = \Delta t \sum_{i=1}^{2M} a_s(i)\left[p_i(x_l) - q_i(1)\right] + u'(x_l,t_s) + g_2(t_{s+1}) - g_1(t_{s+1}) - g_2(t_s) + g_1(t_s), \tag{21}$$

$$u(x_l,t_{s+1}) = \Delta t\sum_{i=1}^{2M}a_s(i)\left[q_i(x_l)-x_lq_i(1)\right]+u(x_l,t_s)+x_l\left[g_2(t_{s+1})-g_2(t_s)\right] \\ +(1-x_l)\left[g_1(t_{s+1})-g_1(t_s)\right], \quad (22)$$

$$\dot{u}(x_l,t_{s+1}) = \sum_{i=1}^{2M}a_s(i)\left[q_i(x_l)-x_lq_i(1)\right]+\dot{g}_1(t_{s+1})+x_l\left[\dot{g}_2(t_{s+1})-\dot{g}_1(t_{s+1})\right]. \quad (23)$$

Using the quasilinearization technique [20] to handle the nonlinearity in equation (1), we have the following scheme

$$\dot{u}(x,t_{s+1})+u(x,t_{s+1})u'(x,t_s)+u'(x,t_{s+1})u(x,t_s)-\nu u''(x,t_{s+1}) = F(x_l,t_s) \\ +u(x,t_s)u'(x,t_s), s=0,1,2,\ldots \quad (24)$$

which leads us from the time layer t_s to t_{s+1}.

Taking the collocation points $x \to x_l$ in equation (24) and using equations (20) - (23), we get

$$\sum_{i=1}^{2M}A(x_l,t_s)a_s(i) = B(x_l,t_s), l=1,2,\ldots,2M, s=0,1,2,\ldots \quad (25)$$

where

$$A(x_l,t_s) = \left\{1+\Delta tu'(x_l,t_s)\right\}q_i(x_l)+\Delta tu(x_l,t_s)p_i(x_l)-\nu\Delta th_i(x_l)-\left\{x_l+x_l\Delta tu'(x_l,t_s) \\ +\Delta tu(x_l,t_s)\right\}q_i(1), \quad (26)$$

$$B(x_l,t_s) = F(x_l,t_s)+\nu u''(x_l,t_s)-u'(x_l,t_s)u(x_l,t_s)-(1-x_l)\dot{g}_1(t_{s+1})-x_l\dot{g}_2(t_{s+1}) \\ -x_lu'(x_l,t_s)\left[g_2(t_{s+1})-g_2(t_s)\right]-(1-x_l)u'(x_l,t_s)\left[g_1(t_{s+1})-g_1(t_s)\right] \\ -u(x_l,t_s)\left[g_2(t_{s+1})-g_1(t_{s+1})-g_2(t_s)+g_1(t_s)\right]. \quad (27)$$

The wavelet coefficients $a_s(i)$, $i=1,2,\ldots,2M$ can be successively calculated from equation (25). This process is started with the initial condition (9). These coefficients are then substituted in equations (20)-(22) to obtain the approximate solutions at different time levels.

IV. NUMERICAL EXAMPLES AND DISCUSSION

In this section, two examples are considered to check the efficiency and accuracy of the Haar wavelet collocation method (HWCM). Cubic spline interpolation is used to find the solution at the specified points. The entire computational work has been done with the help of MATLAB software.

Example 1:

$$u_t+uu_{x_*} = \nu u_{x_*x_*}+\frac{\alpha x_*}{(2\beta t+1)^2}, -1<x_*<1, t>0, \\ u(x_*,0) = \alpha x_*, -1\le x_*\le 1, \\ u(-1,t) = -\frac{A}{2\beta t+1},\ u(1,t) = \frac{A}{2\beta t+1},\ t\ge 0, \quad (28)$$

where $A=\beta+\sqrt{\beta^2+\alpha}$ and $\alpha,\beta>0$ are constants.

Srinivasa Rao [6] showed that the solution of equation (28) for $\nu=1$ and $\alpha>\beta$ is given by

$$u(x_*,t) = \frac{Ax_*}{2\beta t+1}. \quad (29)$$

Let us change the variable $x_*=2x-1$. Now we get the problem,

$$u_t + \frac{1}{2}uu_x = \frac{\nu}{4}u_{xx} + \frac{\alpha(2x-1)}{(2\beta t+1)^2}, 0 < x < 1, t > 0,$$
$$u(x,0) = \alpha(2x-1), 0 \le x \le 1, \quad (30)$$
$$u(0,t) = -\frac{A}{2\beta t+1},\ u(1,t) = \frac{A}{2\beta t+1},\ t \ge 0,$$

and the exact solution becomes

$$u(x,t) = \frac{A(2x-1)}{2\beta t+1}. \quad (31)$$

HWCM solutions are obtained for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 0.2, 0.4, 0.6, 0.8, 1, 2, 3, 4, 5, 6, 8, 10, 11, 13, 15$ with $\Delta t = 0.001$, $M = 32$. The results are compared with the exact solution and are presented in Tables 1-5. Figure 1 represents the HWCM solution for $\alpha = 5$, $\beta = 2$ and $\nu = 1$ at different times t with $\Delta t = 0.001$. Figure 2 is the 3D representation of the HWCM solution for $\alpha = 5$, $\beta = 2$ and $\nu = 1$. In order to measure the accuracy of the solutions obtained by HWCM, we define the error function as

$$\nu(t_s) = \frac{1}{2M}\left\| u(x,t_s) - u_{ex}(x,t_s) \right\| \quad (32)$$

where $u_{ex}(x,t_s)$ is the exact solution (31) at $t = t_s$. The L_2 and L_∞ error norms are calculated for $\alpha = 5$, $\beta = 2$ and $\nu = 1$ at different times t with $\Delta t = 0.001$, $M = 64$ and are presented in Table 9. We observe that the error values are negligibly small which indicate that the Haar wavelet solution is very close to the exact solution.

Figures 3-6 represent the physical behaviour of the HWCM solution for $\alpha = 5$, $\beta = 2$ and $\nu = 0.0001$ at times $t = 0.3, 0.5, 0.8, 1$ with $\Delta t = 0.001$.

Example 2:

$$u_t + uu_x = u_{xx} + \nu\frac{\alpha x}{(2\beta t+1)^2}, 0 < x < 1, t > 0,$$
$$u(x,0) = \sin(\pi x), 0 \le x \le 1, \quad (33)$$
$$u(0,t) = 0, u(1,t) = 0, t \ge 0.$$

The HWCM solutions are obtained for $\alpha = 1$, $\beta = 1$, $\nu = 1$ and $t = 0.1, 0.2, 0.3, 0.4, 0.6, 0.8, 1, 2, 3$ with $\Delta t = 0.001$, $M = 32$. The results are compared with the finite difference method (FDM) solution and are presented in Tables 6-8. Figure 7 represents the HWCM solution for $\alpha = 1$, $\beta = 1$ and $\nu = 1$ at different times t with $\Delta t = 0.001$. Figure 8 is the 3D representation of the HWCM solution for $\alpha = 1$, $\beta = 1$ and $\nu = 1$. Figures 9-10 represents the physical behaviour of the HWCM solution for $\alpha = 5$, $\beta = 2$ and $\nu = 0.19$ at times $t = 0.04, 0.05$ with $\Delta t = 0.0001$.

Table 1: Comparison of HWCM solution and exact solution of Example 1 for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 0.2, 0.4, 0.6$ at different x

x	$t = 0.2$		$t = 0.4$		$t = 0.6$	
	HWCM	Exact	HWCM	Exact	HWCM	Exact
0.1	-2.2226907	-2.2222222	-1.5386331	-1.5384615	-1.1765418	-1.1764706
0.2	-1.6672239	-1.6666667	-1.1540637	-1.1538462	-0.8824450	-0.8823529
0.3	-1.1115603	-1.1111111	-0.7694144	-0.7692308	-0.5883141	-0.5882353
0.4	-0.5557999	-0.5555556	-0.3847181	-0.3846154	-0.2941621	-0.2941176
0.5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.6	0.5557999	0.5555556	0.3847181	0.3846154	0.2941621	0.2941176
0.7	1.1115603	1.1111111	0.7694144	0.7692308	0.5883141	0.5882353
0.8	1.6672239	1.6666667	1.1540637	1.1538462	0.8824450	0.8823529
0.9	2.2226907	2.2222222	1.5386331	1.5384615	1.1765418	1.1764706

Table 2: Comparison of HWCM solution and exact solution of Example 1 for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 0.8, 1, 2$ at different x

x	$t = 0.8$		$t = 1$		$t = 2$	
	HWCM	Exact	HWCM	Exact	HWCM	Exact
0.1	-0.9524159	-0.9523810	-0.8000195	-0.8000000	-0.4444474	-0.4444444
0.2	-0.7143311	-0.7142857	-0.6000255	-0.6000000	-0.3333373	-0.3333333
0.3	-0.4762296	-0.4761905	-0.4000220	-0.4000000	-0.2222256	-0.2222222
0.4	-0.2381173	-0.2380952	-0.2000125	-0.2000000	-0.1111130	-0.1111111
0.5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.6	0.2381173	0.2380952	0.2000125	0.2000000	0.1111130	0.1111111
0.7	0.4762296	0.4761905	0.4000220	0.4000000	0.2222256	0.2222222
0.8	0.7143311	0.7142857	0.6000255	0.6000000	0.3333373	0.3333333
0.9	0.9524159	0.9523810	0.8000195	0.8000000	0.4444474	0.4444444

Table 3: Comparison of HWCM solution and exact solution of Example 1 for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 3, 4, 5$ at different x

x	$t = 3$		$t = 4$		$t = 5$	
	HWCM	Exact	HWCM	Exact	HWCM	Exact
0.1	-0.3076933	-0.3076923	-0.2352945	-0.2352941	-0.1904764	-0.1904762
0.2	-0.2307705	-0.2307692	-0.1764711	-0.1764706	-0.1428574	-0.1428571
0.3	-0.1538472	-0.1538462	-0.1176475	-0.1176471	-0.0952383	-0.0952381
0.4	-0.0769237	-0.0769231	-0.0588238	-0.0588235	-0.0476192	-0.0476190
0.5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.6	0.0769237	0.0769231	0.0588238	0.0588235	0.0476192	0.0476190
0.7	0.1538472	0.1538462	0.1176475	0.1176471	0.0952383	0.0952381
0.8	0.2307705	0.2307692	0.1764711	0.1764706	0.1428574	0.1428571
0.9	0.3076933	0.3076923	0.2352945	0.2352941	0.1904764	0.1904762

Table 4: Comparison of HWCM solution and exact solution of Example 1 for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 6, 8, 10$ at different x

x	$t = 6$		$t = 8$		$t = 10$	
	HWCM	Exact	HWCM	Exact	HWCM	Exact
0.1	-0.1600001	-0.1600000	-0.1212122	-0.1212121	-0.0975610	-0.0975610
0.2	-0.1200002	-0.1200000	-0.0909092	-0.0909091	-0.0731708	-0.0731707
0.3	-0.0800001	-0.0800000	-0.0606061	-0.0606061	-0.0487805	-0.0487805
0.4	-0.0400001	-0.0400000	-0.0303031	-0.0303030	-0.0243903	-0.0243902
0.5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.6	0.0400001	0.0400000	0.0303031	0.0303030	0.0243903	0.0243902
0.7	0.0800001	0.0800000	0.0606061	0.0606061	0.0487805	0.0487805
0.8	0.1200002	0.1200000	0.0909092	0.0909091	0.0731708	0.0731707
0.9	0.1600001	0.1600000	0.1212122	0.1212121	0.0975610	0.0975610

Table 5: Comparison of HWCM solution and exact solution of Example 1 for $\alpha = 5$, $\beta = 2$, $\nu = 1$ and $t = 11, 13, 15$ at different x

x	$t = 11$		$t = 13$		$t = 15$	
	HWCM	Exact	HWCM	Exact	HWCM	Exact
0.1	-0.0888889	-0.0888889	-0.0754717	-0.0754717	-0.0655738	-0.0655738
0.2	-0.0666667	-0.0666667	-0.0566038	-0.0566038	-0.0491803	-0.0491803
0.3	-0.0444445	-0.0444444	-0.0377359	-0.0377358	-0.0327869	-0.0327869
0.4	-0.0222222	-0.0222222	-0.0188679	-0.0188679	-0.0163934	-0.0163934
0.5	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.6	0.0222222	0.0222222	0.0188679	0.0188679	0.0163934	0.0163934
0.7	0.0444445	0.0444444	0.0377359	0.0377358	0.0327869	0.0327869
0.8	0.0666667	0.0666667	0.0566038	0.0566038	0.0491803	0.0491803
0.9	0.0888889	0.0888889	0.0754717	0.0754717	0.0655738	0.0655738

Table 6: Comparison of HWCM solution and FDM solution of Example 2 for $\alpha = 1$, $\beta = 1$, $\nu = 1$ and $t = 0.1, 0.2, 0.3$ at different x

x	$t = 0.1$		$t = 0.2$		$t = 0.3$	
	HWCM	FDM	HWCM	FDM	HWCM	FDM
0.1	0.1170714	0.1170871	0.0510760	0.0510736	0.0240430	0.0240260
0.2	0.2246753	0.2247074	0.0978195	0.0978154	0.0460640	0.0460317
0.3	0.3136929	0.3137420	0.1361537	0.1361489	0.0641715	0.0641271
0.4	0.3758052	0.3758696	0.1625226	0.1625181	0.0767305	0.0766781
0.5	0.4041206	0.4041959	0.1741699	0.1741664	0.0824773	0.0824222
0.6	0.3939934	0.3940719	0.1694115	0.1694094	0.0806166	0.0805641
0.7	0.3439417	0.3440133	0.1478681	0.1478670	0.0708868	0.0708420
0.8	0.2564392	0.2564938	0.1105991	0.1105988	0.0535837	0.0535511
0.9	0.1382517	0.1382812	0.0600829	0.0600829	0.0295319	0.0295148

Table 7: Comparison of HWCM solution and FDM solution of Example 2 for $\alpha = 1$, $\beta = 1$, $\nu = 1$ and $t = 0.4, 0.6, 0.8$ at different x

x	$t = 0.4$		$t = 0.6$		$t = 0.8$	
	HWCM	FDM	HWCM	FDM	HWCM	FDM
0.1	0.0127042	0.0126795	0.0052822	0.0052525	0.0031833	0.0031526
0.2	0.0243892	0.0243423	0.0101866	0.0101301	0.0061527	0.0060943
0.3	0.0340953	0.0340307	0.0143488	0.0142710	0.0086988	0.0086185
0.4	0.0409781	0.0409020	0.0174309	0.0173393	0.0106218	0.0105272
0.5	0.0443575	0.0442773	0.0191324	0.0190360	0.0117351	0.0116357
0.6	0.0437566	0.0436801	0.0192001	0.0191083	0.0118697	0.0117750
0.7	0.0389261	0.0388609	0.0174340	0.0173559	0.0108768	0.0107962
0.8	0.0298516	0.0298041	0.0136921	0.0136353	0.0086307	0.0085721
0.9	0.0167423	0.0167173	0.0078902	0.0078603	0.0050302	0.0049994

Table 8: Comparison of HWCM solution and FDM solution of Example 2 for $\alpha = 1$, $\beta = 1$, $\nu = 1$ and $t = 1, 2, 3$ at different x

x	$t = 1$		$t = 2$		$t = 3$	
	HWCM	FDM	HWCM	FDM	HWCM	FDM
0.1	0.0022514	0.0022205	0.0007622	0.0007312	0.0003926	0.0003616
0.2	0.0043549	0.0042962	0.0014752	0.0014162	0.0007596	0.0007006
0.3	0.0061652	0.0060843	0.0020905	0.0020094	0.0010758	0.0009946
0.4	0.0075418	0.0074467	0.0025611	0.0024657	0.0013171	0.0012216
0.5	0.0083518	0.0082517	0.0028416	0.0027413	0.0014601	0.0013597
0.6	0.0084713	0.0083761	0.0028893	0.0027939	0.0014833	0.0013878
0.7	0.0077880	0.0077070	0.0026642	0.0025830	0.0013664	0.0012852
0.8	0.0062026	0.0061437	0.0021294	0.0020704	0.0010912	0.0010321
0.9	0.0036299	0.0035989	0.0012514	0.0012203	0.0006407	0.0006097

Table 9: L_2 and L_∞ error values of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 1$ at different times t

t	L_2	L_∞	t	L_2	L_∞
0.2	2.49E-05	4.37E-06	6	7.63E-09	1.33E-09
0.4	9.74E-06	1.70E-06	7	4.87E-09	8.48E-10
0.5	6.22E-06	1.08E-06	8	3.29E-09	5.73E-10
0.6	4.12E-06	7.19E-07	9	2.33E-09	4.06E-10
0.8	2.04E-06	3.55E-07	10	1.71E-09	2.97E-10
1	1.14E-06	1.99E-07	11	1.29E-09	2.24E-10
2	1.76E-07	3.06E-08	12	9.96E-10	1.74E-10
3	3.26E-07	6.95E-08	13	7.86E-10	1.37E-10
4	1.06E-07	2.26E-08	14	6.31E-10	1.10E-10
5	1.30E-08	2.26E-09	15	5.14E-10	8.96E-11

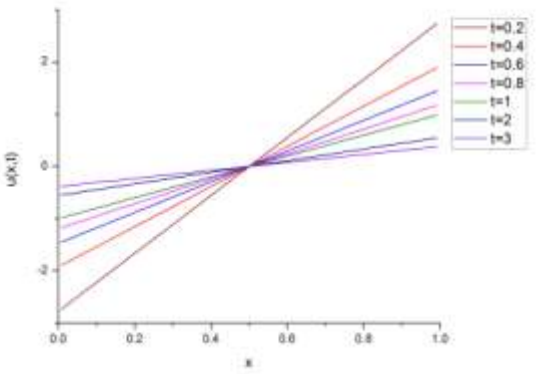

Figure 1: HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 1$ at different times t with $\Delta t = 0.001$

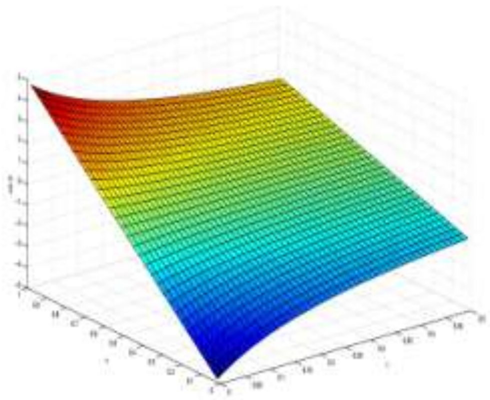

Figure 2: 3D representation of the HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 1$

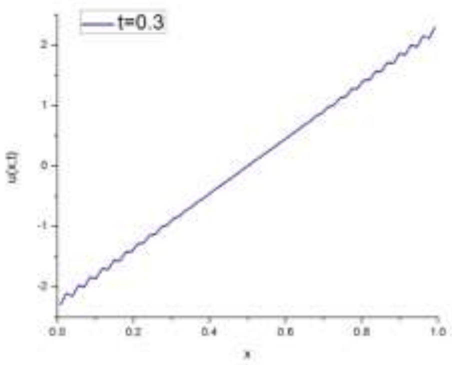

Figure 3: HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 0.0001$ at $t = 0.3$ with $\Delta t = 0.001$

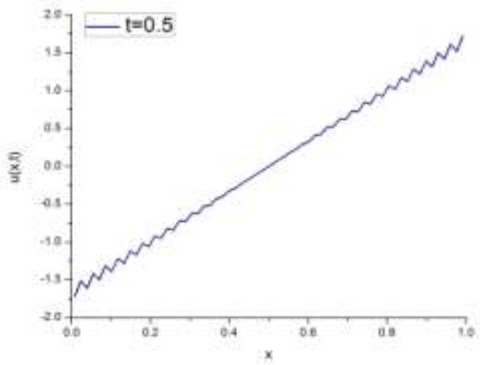

Figure 4: HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 0.0001$ at $t = 0.5$ with $\Delta t = 0.001$

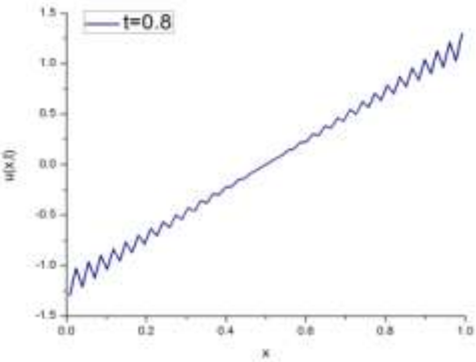

Figure 5: HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 0.0001$ at $t = 0.8$ with $\Delta t = 0.001$

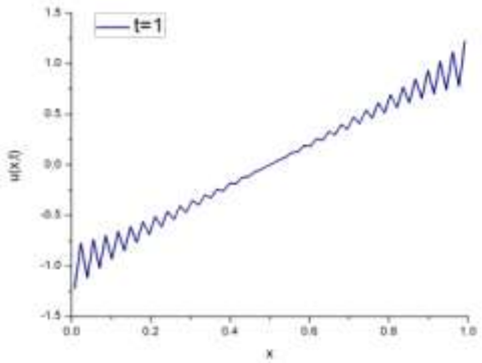

Figure 6: HWCM solution of Example 1 for $\alpha = 5$, $\beta = 2$ and $\nu = 0.0001$ at $t = 1$ with $\Delta t = 0.001$

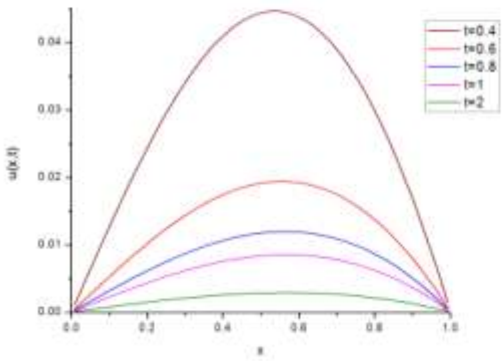

Figure 7: HWCM solution of Example 2 for $\alpha = 1$, $\beta = 1$ and $\nu = 1$ at different times t with $\Delta t = 0.001$

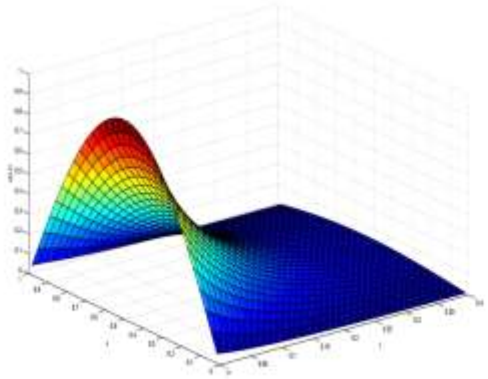

Figure 8: 3D representation of the HWCM solution of Example 2 for $\alpha = 1$, $\beta = 1$ and $\nu = 1$

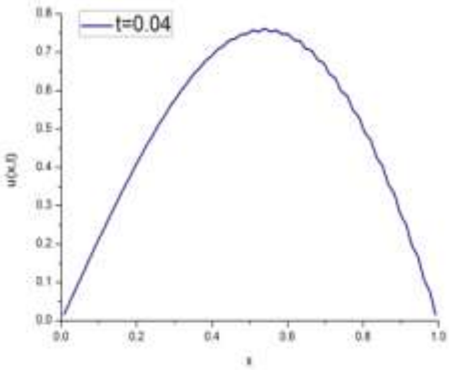

Figure 9: HWCM solution of Example 2 for $\alpha = 1$, $\beta = 1$ and $\nu = 0.19$ at $t = 0.04$ with $\Delta t = 0.001$

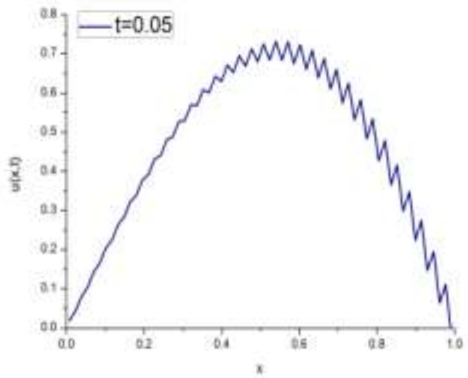

Figure 10: HWCM solution of Example 2 for $\alpha = 1$, $\beta = 1$ and $\nu = 0.19$ at $t = 0.05$ with $\Delta t = 0.001$

V. CONCLUSION

We have solved the non-homogeneous Burgers equation by Haar wavelet collocation method. The solutions obtained are in good agreement with the exact solution and the finite difference method solution. We observe that the error values are negligibly small which indicate that the Haar wavelet solution is very close to the exact solution. Thus the Haar wavelet method guarantees the necessary accuracy with a small number of grid points and a wide class of PDEs can be solved using this approach. This method takes care of boundary conditions automatically and hence it is the most convenient method for solving boundary value problems. Discontinuities in the non-homogeneous Burgers equation, it appears to our knowledge, has not been discussed so far. Such discontinuities which are observed by us are now reported. For linear initial condition, discontinuities are observed for $\nu < 0.0001$ at $t \geq 3$ which are shown in Figures 3-6. Similar behavior is seen in the case of periodic initial condition for $\nu < 0.2$ at $t \geq 0.04$. The discontinuities observed may be attributed as shocks. Further study is going on.

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